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Optical detection of quantum Hall effect of composite fermions and evidence of the $\nu=3/8$ state

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In the photoluminescence spectra of a two-dimensional electron gas in the fractional quantum Hall regime we observe for the first time the states at filling factor $\nu=4/5$, 5/7, 4/11, and 3/8 as clear minima in the emission peak intensity or area. The first three states are described as interacting composite fermions in fractional quantum Hall regime. The minimum in the intensity at $\nu=3/8$, which is not explained within this picture, can be an evidence of a suppression of the screening of the coulomb interaction among the effective quasiparticles involved in this intriguing state.

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Several magnetotransport and magneto-optical studies have been realized on the two-dimensional electron gas (2DEG) created in a modulation-doped semiconductor quantum well, allowing the observation of the integer quantum Hall effect (IQHE) and fractional quantum Hall effect (FQHE).[1–4] Various theories, that envisage the formation of Landau levels (LLs) and composite fermions (CFs), have been proposed to explain these phenomena.[1–5] Many body interactions rule the arrangement of electrons in two dimensions placed in a high magnetic field, leading them to form highly correlated states.[1] The presence of residual disorder is a fundamental ingredient for the observation of these quantum effects. However, FQHE at fractional filling factors that cannot be explained as IQHE of CFs has been observed in transport experiments,[2] and not all the features observed in the photoluminescence (PL) experiments are well understood but they keep on producing sources of research and discussion.[6]

In this work we report the first evidence in optical emission experiments of CFs in fractional quantum Hall regime at filling factors $\nu=4/5$, 5/7, 4/11, and of the exotic state $\nu=3/8$. In particular we analyze the intensity and the area of the 2DEG PL emission as a function of the magnetic field B, for filling factors down to 2/7. Using a model based on the CFs picture[5] and the coulomb screening of the 2DEG (Ref. 7) we explain the structures observed in the spectra at several fractional ν.

The experiments were performed on a 20-nm-thick single sided modulation-doped GaAs/AlGaAs single quantum well with carrier density $n=1.8\times10^{11}$ cm^{-2} and mobility $\mu=1.6\times10^{6}$ cm^{2} V^{-1} s^{-1} measured at 1.5 K. Indium contacts were placed on the sample to allow simultaneous transport and optical experiments. The sample was placed in a dilution fridge and was excited at 1.748 eV with the light of a Ti:sapphire laser. We present the PL measurements at the temperatures of 590 and 35 mK, with laser excitation power lower than 400 μW.

In Fig. 1 we show the PL spectra at 590 mK as a function of B and we observe that the intensity of the emission, deriving from the recombination of the 2DEG electrons in their first Landau level L_0 with photocreated valence holes in their first LL, varies markedly as the magnetic field increases. For filling factors greater than 2, i.e., B smaller than 3.8 T ($B_{\nu=2}$), we observe the emission from the higher LLs, which are at higher energy with respect to the L_0 emission, and a shake-up process at lower energy respect to L_0.[8,9] We focus our analysis on the L_0 emission since its intensity shows clear minima at specific values of the magnetic field, and in Fig. 2 (top panel) we plot the intensity of the PL peak as a function of B. In the measurement at the temperature of 590 mK we observe a strong suppression of the PL intensity at $\nu=1$ ($B=7.6$ T). This suppression has been observed and studied by other authors with continuous wave[3] and time resolved PL.[7] Another striking feature is the big step at $\nu=2$ at $B\sim3.8$ T. We can see that at this magnetic field the transitions from the second electron LL, i.e., L_1, disappear. This is due to the emptying of this LL since the Fermi level drops to the first one, allowing the identification of $\nu=2$. Therefore we get the value of 7.6 T for $\nu=1$, in very good

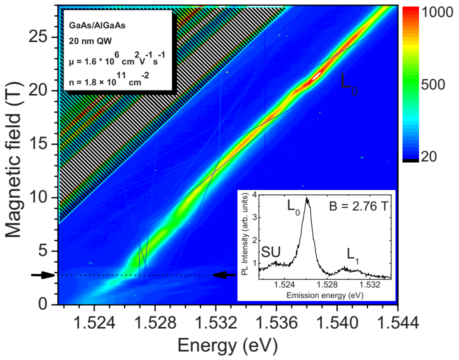

FIG. 1. (Color online) Contour plot representing the emission spectra of a 2DEG in a modulation-doped GaAs/AlGaAs 20-nm-thick QW with carrier density $n=1.8\times10^{11}$ cm^{-2} and mobility $\mu=1.6\times10^{6}$ cm^{2} V^{-1} s^{-1} as a function of magnetic field measured at $T=590$ mK. In the inset is reported one representative spectrum taken at $B=2.76$ T showing the emission from the shake-up process (SU) and from Landau levels (L_0 and L_1). The region in the upper left corner corresponds to the emission from the bulk GaAs, which is not relevant to this work.

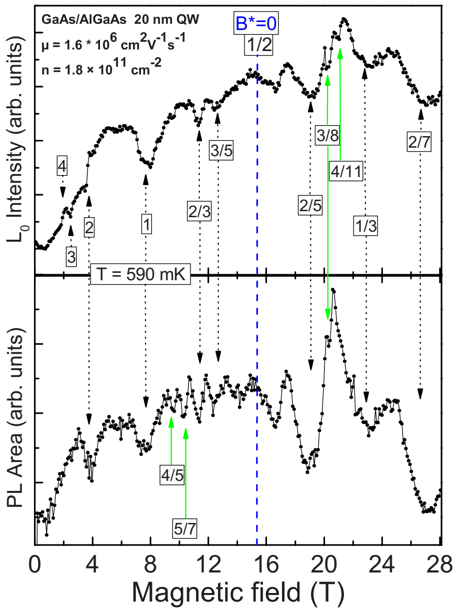

FIG. 2. (Color online) Intensity and area of the emission at $T=590$ mK. Electrons from 2DEG in L_0, recombining with photocreated valence holes in L_0, generate the PL peak whose intensity is shown in the top panel. The integrated area of the total emission from the 2DEG is shown in the bottom panel. Suppressions of the intensity and area in correspondence of the fractional filling factors indicated in boxes are clearly observable.

agreement with a decrease in the PL intensity at this $\nu=1$ filling factor. Reducing the magnetic field from this value we observe a PL intensity decrease until the value at which the emission from the higher LLs starts to be resolved from the L_0. On this monotonic decrease in intensity we observe a weak minimum at $\nu=3$ and a small step at $\nu=4$, a behavior that has been also observed by Byszewski and co-workers.[10] These observations suggest that the intensity follows the pattern *minima-step-minima-step* for *odd-even-odd-even* integer filling factors. Since we are observing the emission from the first LL, we should expect that for magnetic field lower than $B_{\nu=2}$ the emission would be very low since for all that range the L_0 is fully occupied. This is what was actually observed by Turberfield *et al.*[4] The behavior of the PL of our sample agrees with the one observed by Turberfield but shows additional features, i.e., the minimum at $\nu=3$ and the step at $\nu=4$ related to the influence by the electrons in the higher LLs on the emission from L_0. The presence of a step instead of a minima at $\nu=4$ (as well as at $\nu=2$) is due to the fact that at this filling factor the emission from L_2 extinguishes since it is completely empty. This situation is similar to the one at $\nu=2$ when the emission from L_1 vanishes since it drains and the summation of the contributes of the higher LLs changes the step into a minimum. The behaviors at $\nu=3$ and $\nu=4$ are analogous to the ones at $\nu=1$ and $\nu=2$; both of them are observed in the L_0 emission, but the phenomena at $\nu=3$ and $\nu=4$ involve the L_1 (instead of L_0). These features would be expected also in the emission from L_1 close to $\nu=3$ and $\nu=4$, however the feeble intensity from L_1 does not allow to resolve them in our measurements.

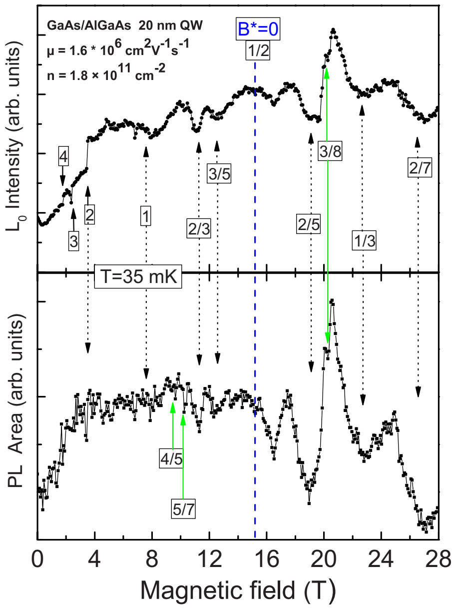

FIG. 3. (Color online) Intensity (top panel) and area (bottom panel) of the PL emission at T=35 mK.

At higher fields when $\nu<1$, the emission is suppressed at several values of the magnetic field. We identified some of these values as corresponding to $\nu=2/3$, 3/5, 2/5, 1/3, and 2/7. These fractions agree with the ones expected by the theory of CFs.[5,11] Within this model, the FQHE for the electrons, observed at these fractional fillings also in the transport measurements,[1] is seen as the IQHE of quasiparticles, the CFs, which originate from the mutual interaction of the electrons in the magnetic field. These quasiparticles are formed by the bounding of an electron with an even number of magnetic flux quanta. For example, at magnetic fields close to $\nu=1/2$, CFs constituted by an electron with two flux quanta are formed (^{2}CF), while at $\nu=1/4$ with four flux quanta (^{4}CF). The net result is the passage from a system of interacting electrons in a magnetic field B to another of vanishing interacting CFs in a residual magnetic field B^*. This B^* is equal to zero at filling factor $1/n$ with n an even integer. The change of the magnetic field from B^* leads to the formation of LLs for these CFs, which indeed are fermions. The relation between the electron filling factor ν and the CF filling factor ν_{2p} is $\nu=\nu_{2p}/(2p\nu_{2p}\pm 1)$, in which p is an integer number and the minus has to be taken when the B^* is antiparallel respect to B. We obtain series of fractions symmetric respect to the $B^*=0$ value which include $\nu=2/3$, 3/5, 2/5, 1/3, and 2/7.

In Fig. 3 we report the PL intensity and area at the temperature of 35 mK. One notable difference is that the deep minimum at $\nu=1$ observed at 590 mK changes its shape at the lower temperature. At 35 mK we observe a wide valley around $\nu=1$ with a very narrow minimum which comes out close to 7 T and additional experiments are required in order to explain this behavior. An expected minimum at $\nu=3/7$, the symmetric of $\nu=3/5$ with respect to B^* and obtainable from the aforementioned formula with $\nu_2=3$, is not resolved, since lies within a deep minimum centered at $\nu=2/5$.

Minima in the PL intensity and area are also present on both side of $\nu=1/2$, of which the one at right side is more evident, either at 590 and 35 mK. A possible reason for the presence of these wide minima is the manifestation of the IQHE of composite fermions for $\nu_2 > 3$. Indeed upon approaching $B^* = 0$ there is a thickening of states corresponding to consecutive filling of CF Landau levels, so that it is progressively more difficult to resolve the individual minima while the overall effect is a gradual diminution of the PL intensity moving away from $B^* = 0$.

To give a physical interpretation of the B-dependence of the PL intensities, we calculate the area under the PL curves (bottom panels in Figs. 2 and 3). For magnetic fields at which the emissions from the L_0 and the higher LLs are simultaneously present, we considered the area of the whole emission from the 2DEG, including that from higher LLs and the shake-up process. However for weak magnetic field, i.e., $B < B_{\nu=2}$, the calculation of the area is very difficult since the 2DEG emission is very close to the tail of the emission from bulk GaAs. For this reason we will focus our attention to the high magnetic field regime, i.e., for $B > B_{\nu=2}$. We can observe that the PL area does not always exhibit minima at the same magnetic fields than the ones in the intensity. Furthermore some features, that are weak in the graph of the intensity of the PL peak, in the graph for the area are much more evident.

As regards the PL efficiency, the integrated PL emission is proportional to the inverse of the recombination time τ_r of the photocreated holes with the electrons of the 2DEG. The recombination time, that is a measure of the probability of the $e-h$ recombination, is related to the overlap between the electron and hole wave-functions. According to previous works,[4,7] this change in the wave-functions overlap depends on the Coulomb screening efficiency of the 2DEG and can be separated in two spatial components: (1) the overlap of the wave-functions along the direction of growth of the sample z and (2) the overlap in the plane xy normal to z.

(1) Our quantum well is asymmetric and the 2DEG and the photocreated valence holes are separated by a distance of the order of the QW width. We can model the high mobility and high density 2DEG as a metal sheet and the valence hole as a positive charge approaching this surface. The result is that the electrons screen the coulomb potential of the photocreated hole creating an imaginary negative charge that attracts the hole increasing the overlap. This is true as long as the electrons can scatter in empty states without changing their energy and takes place in the LL which is partially occupied. Otherwise if all the LLs are fully occupied there are no empty states available for the electrons and they cannot correlate their positions to screen efficiently the hole potential. So when the LLs are fully occupied the PL intensity is suppressed since the screening is diminished.

(2) The silicon remote ionized impurities create a random potential in the xy plane. The 2DEG can screen this potential when the LLs are partially empty. When the LLs are fully occupied the screening is not effective and the result is that electrons and holes tend to localize at different places of the random potential landscape because of their different charge (around valleys or hills). This causes a reduction in the overlap and the suppression of emission intensity.

This analysis has been used to give an explanation for the suppression at $\nu = 1$ and for the behavior in the IQH regime.[4,7] The fact that the CFs form LLs, in analogy to what happens for electrons, makes possible to extend this interpretation to the FQHE PL intensity minima such as the ones at $\nu = 2/3$, 3/5, 2/5, 1/3, and 2/7.

Observing carefully Fig. 2 we can notice that there are extra minima and that they don't belong to the IQHE of CF series. These minima correspond to the filling factors $\nu = 4/5$, 5/7, 4/11, and 3/8. These fractions have been clearly observed in recent transport measurements[2] creating interesting discussions on their origin but, to our knowledge, they have not been observed up to now by optical techniques. At 590 mK, the quantum Hall liquid states at $\nu = 4/11$ and 3/8 are probably not stable, as the energy gaps of these two fractions are very small. However, the FQHE correlation may still be effective even at this temperature.

Recent PL experiments carried out on samples with mobility between 1×10^6 $\mathrm{cm^2/Vs}$ and 3×10^6 $\mathrm{cm^2/Vs}$ and with intermediate electron density (of about $\sim 10^{11}$ $\mathrm{cm^{-2}}$), show that in this regime the interplay between disorder and Coulomb interactions give rise to interesting interaction related effects.[12] Our sample is in this mobility and density regime ($\mu = 1.6 \times 10^6$ $\mathrm{cm^2/Vs}$, $n = 1.8 \times 10^{11}$ $\mathrm{cm^{-2}}$) and we suppose that these interaction related effects and the specificity of PL can favor the observation of the state $\nu = 4/5$, 5/7, 4/11, and 3/8 in the optical spectra.

Let us note that the minima at $\nu = 4/5$ and $\nu = 5/7$ lie between electron filling factors 1 and 2/3, while the ones at $\nu = 4/11$ and $\nu = 3/8$ lie between $\nu = 2/5$ and $\nu = 1/3$ (placed on a steep slope of the intensity curve). These two regions correspond to the one between CF filling factors $\nu_2 = 1$ and $\nu_2 = 2$, where the subscript 2 in ν_2 indicates that it refers to the ^{2}CF. To explain these minima in terms of CFs we can introduce an analogy with electrons. In the transport measurements between $\nu = 1$ and $\nu = 2$ we observed plateaus in the transversal resistance R_{xy} and minima in the longitudinal resistance R_{xx} at the filling factors 4/3 and 5/3, according to the findings of several authors.[1] These features can be interpreted, within the CF theory, as, respectively, the plateaus and minima associated to the ^{2}CF states at $\nu_2 = 1$ and $\nu_2 = 2$, which emanated from $\nu = 3/2(1+1/2)$ instead of $\nu = 1/2$, so that the state $\nu = 3/2$ results as the analogous of the $\nu = 1/2$ in the upper level of the LL_0. When not all the electrons lie in the lowest level of the LL_0, the electrons of the full levels can be neglected since their energy is far from the Fermi level; therefore according to the model of Jain,[5] the flux quanta have to be attached only to the electrons in the partially filled level. The relation that links the filling factors has to be modified in $\nu = N + \nu_{2p}/(2p\nu_{2p} \pm 1)$, with N equal to the number of full levels (i.e., $3/2 = 1 + 1/2$ and $N = 1$).[13] In order to apply this model to explain the minima at $\nu = 4/5$ and $\nu = 5/7$ we can imagine them as FQHE of CFs. This means that CF-CF interaction can't be assumed as vanishing, but the CFs are expected to be interacting and the residual interaction manifests itself as FQHE. An observation of this residual interaction has been done by Gallais *et al.*[14] in inelastic light scattering experiments in the range $2/5 < \nu < 2/7$; in this range of B the states at $\nu = 4/11$ and $\nu = 3/8$ have been specifically observed only in transport experiments. These states, detected in our optical experiments, are evidenced in Fig. 4 where the curves showing the PL intensity and area

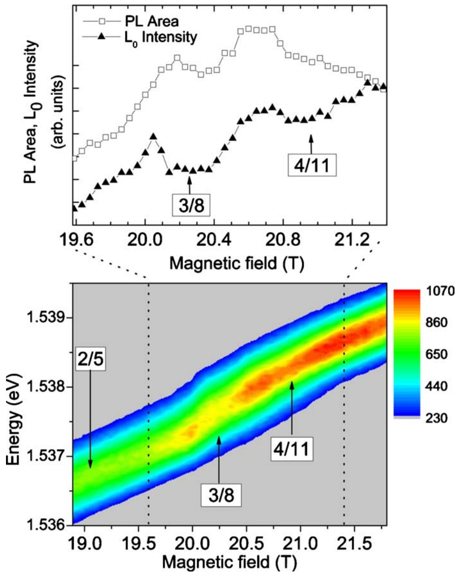

FIG. 4. (Color online) Contour plot evidencing the PL suppressions at $\nu=3/8$ and 4/11 (bottom panel), at $T=590$ mK. The $\nu=2/5$ is shown as a reference. The curves in the top panel show the PL intensity and area in the region around $\nu=3/8$ and 4/11 with resolution of $\Delta B=0.046$ T.

close to $\nu=3/8$ are reported. This CF-CF interaction implies composing CFs of CFs, which is a little bit complicated because of the attachment of additional flux quanta to the CFs. So a ^{2}CF become a ^{4}CF (actually the states interpreted as IQHE of ^{4}CF around $\nu=1/4$ can be equally understood as FQHE of ^{2}CF). In analogy the states at $\nu=4/5$ and $\nu=5/7$ result to be the $\nu_2=4/3$ and $\nu_2=5/3$ states of CFs emanating from $\nu=1/2$. In this picture, two more flux quanta have to be bound to the CFs of the partially filled level, obtaining a mixed states of ^{2}CF (in the completely full CF level) and ^{4}CF. After this transformation, $\nu_2=4/3$ and $\nu_2=5/3$ can be imagined, respectively, as the $\nu_4=1$ and $\nu_4=2$ emanating from $\nu_2=3/2$ (that corresponds to $\nu=3/4$).[15] The same happens for $\nu=4/11$ ($\nu_2=4/3$), but not for $\nu=3/8$. The origin of $\nu=3/8$ can neither be explained as IQHE nor as FQHE of CFs. In fact $\nu=3/8$ corresponds to $\nu_2=3/2$, a fractional value that doesn't belong to anyone of these series.

We analyzed the PL line shape as function of B and we observed that the relation between the line width, the deepness of minima in the PL intensity and in the area is not univocal. As regards the two fractions 4/5 and 5/7, the minimum is present mainly in the PL area and less marked in the intensity. This finding reveals that at these fractions a line narrowing takes place, which can be tentatively attributed to a modification of the PL lifetime and of the localization when the interacting composite fermions undergo the fractional quantum Hall regime.

Recent works[6,16] give insight on the correspondence between the observed PL emission shape at fractional ν and the energy spectrum, i.e., binding energies vs momentum, of the initial photoexcited system and of the final state after the recombination. Since the inhibition of the recombination is caused by localization and change in the screening response of the electron gas,[7] the smaller reduction in the PL intensity for the 4/5 and 5/7 states with respect to the adjacent 2/3 state can be attributed to the fact that the energy spectra of the initial or final states and the localization and screening response of the charges are more modified when the CFs undergo IQHE rather than FQHE. As regards the 4/11 state, it is hardly resolvable in the curves of the PL area either at 590 mK and at 35 mK (bottom panel of Fig. 2 and 3, respectively) since the fraction falls in a region with a cusp and rapid variation in the PL area, between the strong minima of the 2/5 and 1/3 states. In particular at 35 mK the shrinkage of this cusped hampers the development of the minimum at 4/11.

Finally, we want to notice that the 3/8 state has been observed only in few magnetotransport experiments as a small minimum in R_{xx},[2] its enigmatic origin is still under debate and several theories has been proposed to explain it.[17–19] In the CFs picture the $\nu=3/8$ can be viewed as a state with the lowest level of LL_0 of CF completely occupied by ^{2}CF and the second level half-occupied by ^{2}CF. These ones, bound with two additional magnetic flux quanta, form a Fermi sea of ^{4}CF. Within this model, the minima in PL intensity and in R_{xx} could be explained by a mechanism which foresees an instability in the Fermi sea with opening of a gap, i.e., involving a *p*-wave pairing of composite fermions in the spin reversed sector.[18] This pairing is similar to the one which has been proposed to explain the minimum in R_{xx} observed at $\nu=5/2$, due to the occurrence of a Bardeen-Cooper-Schrieffer-like state, called the Pfaffian state.[20–23] A different approach, invoking the grouping of quasiparticles rather than a multiflavor CF picture, has also been proposed.[18] A third picture invokes the clustering of composite bosons that carry an odd number of flux quanta that can be positive as well as negative.[19] In the framework of all these theories, that properly describe the occurrence of a minimum in R_{xx} at $\nu=3/8$, the optical observation of the 3/8 state in the optical emission could be a cue for a better understandings of its nature.

In conclusion, we provide the first evidence in optical experiment of the states at filling factor $\nu=4/5$, 5/7, 4/11, and 3/8 as suppression of the PL emission. Using a model based on the CFs picture[5] and the coulomb screening of the 2DEG (Ref. 7) we explain the structures observed in the spectra at several fractional ν. The minimum observed in the PL intensity at $\nu=3/8$ cannot be described in the framework of the CFs model and suggests the weakening of the screening of the coulomb interaction among the quasiparticles which compose this exotic state.

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